# Incorporating Hysteresis at the Grain Scale of a Multi-Scale Material Model

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This paper proposes a multi-scale energy based material model for poly-crystalline materials. The model considers the poly-crystalline, grain and crystal scales, exhibits magnetostriction and hysteresis and is merely based on a set of physical constants. The model is verified with existing measurement data for different stress levels and is found to provide a good accuracy.

Index Terms-Magnetostatics, nonlinear magnetics, magneto-mechanical couplings, hysteresis loops, multi-scale modeling.

# I. INTRODUCTION

**▼**ONTEMPORARY electrotechnical design requires highly accurate models for ferromagnetic materials which are capable of representing hysteresis and magnetostriction effects properly. This is possible using energy based material models such as the Armstrong model [1] or the multi-scale model [3], when accomplished with a hysteresis model. The Armstrong model uses an incremental hysteresis model for a single-crystal material [2]. The multi-scale model incorporates Hausers' hysteresis model for poly-crystalline materials [4], [7]. In this paper, however, the hysteresis effect is considered at the grain scale. In contrast to [6], where artificial thresholds prevent early switching of magnetic domains, here, the grain scale magnetization of the previous time step is used for correcting the Boltzmann distribution. The new approach requires less parameters compared to the Preisach, Jiles-Atherton and Hauser models and does not demand a tedious parameter identification work, since the model is merely based on physical constants which can be looked up in standard references. Moreover, the proposed approach considers the magneto-mechanical coupling.

## II. VARIOUS SPATIAL MATERIAL SCALES

The material model uses three material scales (Fig. 1): (i) the poly-crystal scale, which is a representative volume element consisting out grains with different sizes and orientations; (ii) the grain scale, consisting of several crystals which are almost perfectly aligned to each other; (iii) the crystal scale, consisting of one perfect crystal. The poly-crystal scale takes the external magnetic field  $H_{\rm ext}$  and the external mechanical stress  $\sigma_{\rm ext}$  as inputs. The model parameters are a few physical constants and a user-defined accuracy. The initial conditions are the grain orientations and initial guesses for the mean poly-crystal scale/ grain scale magnetizations  $M_{\rm m}/M_{\rm g}$  and magnetostrictive strains  $\epsilon_{\rm m}/\epsilon_{\rm g}$ , respectively. The initial grain

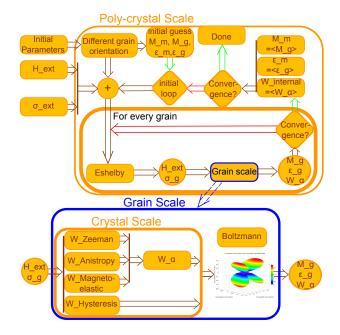


Fig. 1. Flowchart of the material model

orientations are obtained as proposed in [8], leading to the grain scale stresses  $\sigma_{\rm g}$  by solving the Eshelby inclusion problem [3], [5], [10]. Homogeneous elastic properties and a uniform strain are assumed in the grain representing the inclusion.

 $H_{\rm ext}$  and  $\sigma_{\rm g}$  are the inputs for the crystal scale, allowing to calculate the three standard energy functions [1], [3] and a new energy function modelling hysteresis (see Section II-A).

The Boltzmann distribution on the energy functions leads to the grain scale magnetizations  $M_{\rm g}$  and magnetostrictions  $\epsilon_{\rm g}$  (see Section II-B). When  $M_{\rm g}$  and  $\epsilon_{\rm g}$  converged, a weighted average procedure is performed, according to the Eshelby inclusion problem [3], [5], [10]. The procedure is repeated until convergence is reached at the poly-crystal scale.

### A. Crystal scale

The energy functions calculated at the crystal scale are [1], [3]: (i) the Zeeman energy  $W_{\rm H}$  based on the saturation magnetization  $M_{\rm S}$ , (ii) the magneto-crystalline anisotropy energy  $W_{\rm an}$  based on the crystal anisotropy constants  $K_1$  and  $K_2$  and (iii) the stress induced anisotropy energy  $W_{\sigma}$  based on saturation magnetostrictions  $\lambda_{100}$  and  $\lambda_{111}$ . The additional energy function modelling hysteresis is described as:

$$W_{\text{hys}} = -\mu_0 \frac{M_{\text{s}}}{\chi_0} \left( \alpha_{\text{M1}} M_{\text{hys1}} + \alpha_{\text{M2}} M_{\text{hys2}} + \alpha_{\text{M3}} M_{\text{hys3}} \right)$$
(1)

This energy function stands alone (see Fig. 1) and will increase the obtained anhysteresis probability, closest to grain scale magnetization in the previous time step  $M_{\rm hys}$  (See Section II-B). Here,  $\chi_0$  is the grain scale magnetic susceptibility which is not measured as a single poly-crystal constant [3], but simulated for every grain in the first two time steps.

#### B. Grain scale

Due to crystal defects, not all domains have the same orientation, even in a single grain. This is taken into account by the Boltzmann distributions  $P_{\alpha}$ ,  $P_{\text{hys}}$  and  $P_{\text{g}}$ :

$$P_{\alpha} = \frac{\exp(-A_{\rm s}(W_{\rm H} + W_{\rm an} + W_{\sigma}))}{\int_{\alpha} \exp(-A_{\rm s}(W_{\rm H} + W_{\rm an} + W_{\sigma}))}$$
(2)

$$P_{\rm hys} = \frac{\exp(-\beta A_s W_{\rm hys})}{\int_{\alpha} \exp(-\beta A_s W_{\rm hys})} \quad (3) \qquad P_{\rm g} = \frac{P_{\alpha} P_{\rm hys}}{\int_{\alpha} P_{\alpha} P_{\rm hys}} \quad (4)$$

The standard Boltzmann distribution  $P_{\alpha}$  includes the parameter  $A_s = (3\chi_{m0})/(\mu_0 M_s^2)$  as a measure for the regularity of the crystal. In this paper, the poly-crystal scale magnetic susceptibility  $\chi_{m0}$  is replaced by the grain scale susceptibility  $\chi_0$ . The additional energy function (1), in the crystal scale,

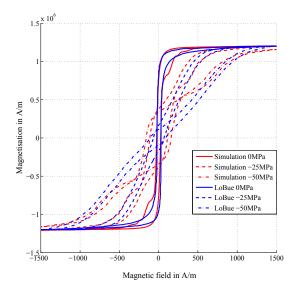


Fig. 2. Comparison between the results from the model, indicated in red, and the reference results reported in [9], indicated in blue.

models the hysteresis by increasing the obtained anhysteretic probability (2) (using the three standard energy functions), closest to  $M_{\rm hys}$ . An additional Boltzmann distribution  $P_{\rm hys}$  with a correction factor  $\beta$  (here set to 1.2, 1 and 0.9 for respectively 0, -25 and -50 MPa) is introduced. The correction factor  $\beta$  models the pinning effect, as suggested by Hauser [7], which was not considered for obtaining  $A_{\rm s}$  in [3]. The combination of both (see (4)) allows to predict  $M_{\rm g}$  and  $\epsilon_{\rm g}$  as in [1] and [3], but with inclusion of the hysteresis effect. The method is valid for alternating as well as for rotating magnetic fields.

#### III. VALIDATION

The simulated results are compared to measurements on Fe-Si3% reported in [9]. Fig. 2 shows that the results from the model, indicated in red, are in good agreement with the reference results, indicated in blue. The remaining differences are attributed to unknown internal stresses, possibly introduced by the manufacturing process.

# IV. CONCLUSION

An energy based material model is extended by a hysteresis model inserted at the grain scale. Although based on a small set of parameters, the extended model shows a good agreement for a poly-crystalline material with this approach it's possible to link micro- and macroscale effects by a reliable model.

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